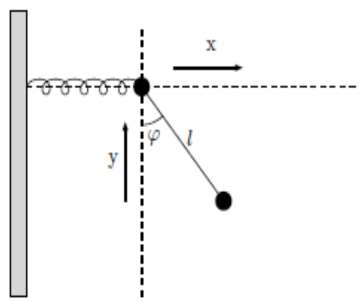
**PHY 711: ANALYTICAL DYNAMICS**

## Additional Practice Problems I

### Problem 1

A point mass $m_1$ is at the end of a horizontally placed massless spring, so that it can undergo oscillations along a horizontal line. A pendulum of length $l$ is now suspended from the mass $m_1$, as shown in figure. The bob of the pendulum has mass $m_2$.
a) Obtain the Lagrangian and the equations of motion for the system. b) Simplify the equations of motion for small amplitudes.
c) Obtain the normal modes and eigenfrequencies of the oscillations.




### Problem 2

A thin massless open ended tube AB of length $L$, revolves with a constant angular velocity $\omega$ around the vertical axis CA, keeping a fixed angle $\alpha$ with it. A bead of mass $m$ can slide inside the tube (neglect friction). See figure for a sketch of the geometry of the set-up.
a) Write down the Lagrangian for this system and determine the equation of motion of the bead.
b) Determine the bead's motion if at $t = 0$ the bead is at a distance $a$ away from point A and its initial velocity along the tube is zero.
c) What is the minimum value of the angular velocity such that the bead remains at an equilibrium position inside the tube?

### Problem 3

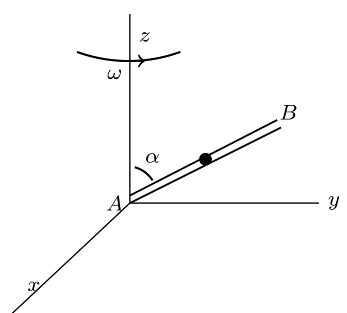
A particle moves in a circular orbit in a force field which is radial and given by $F = -k/r^2$, where $k$ is a positive constant, $k > 0$. If suddenly the value of $k$ is reduced to half its original value, show that the particle's orbit becomes parabolic.

## Problem 4

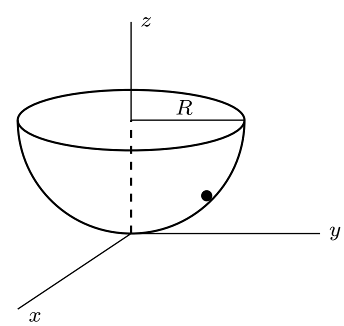
A particle moves in a central force field given by $F = kr$, where $k$ is again positive. (This means that the force is repulsive.) Obtain the orbit of the particle, $r$ as a function of $\varphi$, the angular variable. (*Hint*: The substitution $u = 1/r^2$ will be helpful for doing the integral.)

## Problem 5

A particle can slide without friction on the inner surface of a hemispherical bowl (of negligible thickness) which is resting on the ground as shown. The radius of the bowl is $R$. Obtain the Lagrangian and equations of motion of the particle. (You should keep in mind that the particle can have angular motion as well as radial motion. The particle cannot get off the surface, so that the vertical motion is related to the radial motion.)



Problem 2



Problem 5

## Problem 6

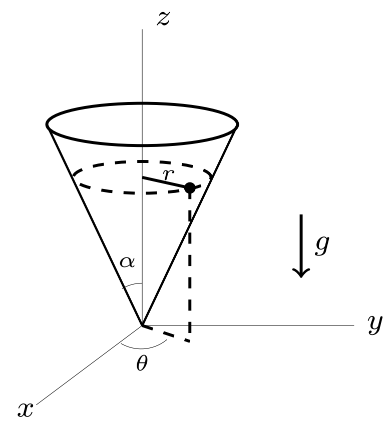
A planet is an elliptical orbit around the Sun, the orbit being described by

$$\frac{1}{r} = \frac{1 + \epsilon \cos\varphi}{a\,(1 - \epsilon^2)}$$

where $a$ is the semi-major axis and $\epsilon$ is the eccentricity. You may also recall that $\mu\, r^2 \dot{\varphi} = l$. The planet has the maximum speed $v_{\text{max}}$ at perihelion and minimum speed $v_{\text{min}}$ at aphelion. Calculate the ratio $v_{\text{max}}/v_{\text{min}}$ from the information given. Hence obtain the eccentricity of the orbit in terms of this ratio. (This is in fact one way of determining $\epsilon$ for planets. The relevant speeds can be measured, in principle, using the Doppler shift of spectral lines when the planet is at perihelion and aphelion.)

## Problem 7

A particle of mass $m$ moves on the inner surface of a cone with opening angle $\alpha$, which is placed vertically upright on the floor with its apex touching the floor, see figure.

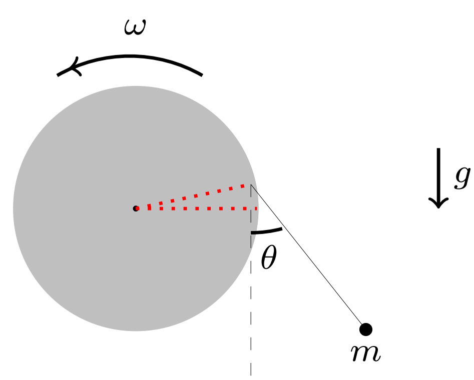



Obtain the Lagrangian and the equations of motion for the motion of the particle. Ignore friction. (*Hint: Cylindrical coordinates might be best. Keep in mind that there is motion corresponding to change in angle, radius and height, but not all are independent.*)

## Problem 8

A wheel of radius $R$ is rotating in the vertical plane with angular velocity $\omega$. From the rim of the wheel is suspended a pendulum of length $l$, with a bob of mass $m$. (There is a little massless axle at the point of suspension, so the string of the pendulum *will not* wind around the wheel.)

a) Obtain the Lagrangian and the equations of motion for the pendulum.




## Problem 9

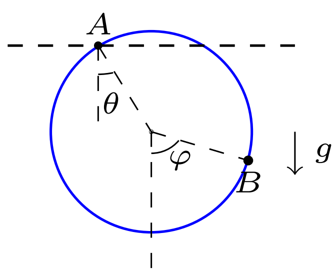
A hoop of uniform density and mass $M$ and radius $R$ (shown in blue in figure) is pivoted at a point (shown as $A$) on the circumference and can oscillate like a pendulum, in the vertical plane. A bead of mass $m$ (shown as a black dot, point $B$) can slide frictionlessly on it.

a) Obtain the Lagrangian and the equations of motion.

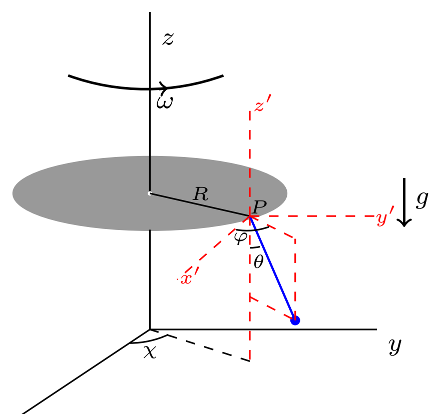
b) For small oscillations of the hoop and the bead, (i.e., for small values of $\theta$, $\varphi$ in figure), find the frequencies for the normal modes of the system. (*Hint: First consider a small element of the hoop to work out its kinetic energy.*)

**Problem 10**

There is a disk of radius $R$ which is rotating with angular velocity $\omega$ in the horizontal plane (say, the $(x, y)$-plane) around a vertical axis, see figure. A pendulum, with a string of negligible mass and a bob of mass $m$, is hung from a point $P$ on the edge of the disk and can undergo oscillations, not necessarily confined to a vertical plane. Find the Lagrangian and equations of motion for the bob of the pendulum. (*Hint: Take the coordinates of the point of suspension of the pendulum on the disk as* $(X, Y, Z) = (R\cos\chi, R\sin\chi, h)$ *where* $h$ *is fixed and the coordinates of the bob as* $(x', y', z')$ *in terms of the red dashed coordinates and write them in terms of angles shown. Then add them vectorially to get the coordinates* $(x, y, z)$.)




Problem 9




Problem 10